Soft-gluon effective coupling and cusp anomalous dimension

Massimiliano Grazzini
University of Zurich

in collaboration with Stefano Catani and Daniel de Florian
+ Simone Devoto and Javier Mazzitelli

Radcor 2019, Avignon, September 9th 2019

Universität Zürich[UZH]

FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

Outline

- Introduction
- Soft coupling at NLL and cusp anomalous dimension
- Soft coupling to all orders
- Second order result to all orders in ε
- Relation with the cusp anomalous dimension at the conformal point
- Generalised Casimir scaling
- Summary

Introduction

Hard scattering observables are sensitive to soft-gluon effects

When the cancellation of real and virtual contribution is unbalanced large logarithmic terms may arise $\alpha_S^n L^{2n}$ that may invalidate the perturbative expansion

At the lowest order:

Intensity of soft-gluon radiation

$$dw_i^{DL} = C_i \frac{\alpha_S}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2} \simeq C_i \frac{\alpha_S}{\pi} \frac{dz}{1-z} \frac{dq_T^2}{q_T^2}$$

$$C_i = C_F \qquad i = q, \bar{q}$$

$$C_i = C_A \qquad i = g$$

Double logarithmic spectrum

Examples:

Small transverse momenta: $\ln Q^2/p_T^2$

Threshold: $\ln(1 - Q^2/s)$

→ **Resummation needed**

Introduction

Resummation is typically organised through the product of hard and Sudakov factors

- Hard factor: computable at fixed order (no large log)
- Sudakov factor: exponentiate the large logarithmic terms and is obtained up to NLL through the integration of a universal kernel

Bonciani, Catani, Mangano,Nason (2003)
Banfi, Salam, Zanderighi (2005)

Dominant part of Sudakov form factor obtained by the use of the CMW kernel

Catani, Marchesini, Webber (1991)

Additional NLL contributions in the Sudakov form factor may arise from

- Soft emissions at large angles
- Collinear (non soft) emissions

Soft coupling at NLL

NLL resummation achieved through the following replacement

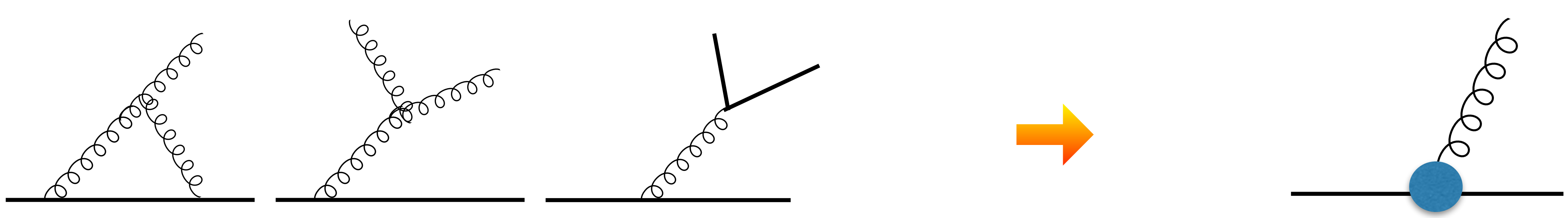
$$C_i \frac{\alpha_S}{\pi} \to \mathcal{A}_i^{CMW}(\alpha_S(q_T^2)) = C_i \frac{\alpha_S^{CMW}(q_T^2)}{\pi} = C_i \frac{\alpha_S(q_T^2)}{\pi}\left(1 + \frac{\alpha_S(q_T^2)}{2\pi} K\right)$$

- QCD coupling evaluated at the scale q_T of the soft gluon (LL)
- $\mathcal{O}(\alpha_S^2)$ correction included proportional to the coefficient K

Soft-gluon effective coupling to NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_F$$

Replacement in the exponent of the Sudakov form factor: produced by the **correlated** emission of soft partons (gluons and $q\bar{q}$ pairs)



Soft coupling at NLL

NLL resummation achieved through the following replacement

$$C_i \frac{\alpha_S}{\pi} \to \mathcal{A}_i^{CMW}(\alpha_S(q_T^2)) = C_i \frac{\alpha_S^{CMW}(q_T^2)}{\pi} = C_i \frac{\alpha_S(q_T^2)}{\pi}\left(1 + \frac{\alpha_S(q_T^2)}{2\pi}K\right)$$

- QCD coupling evaluated at the scale q_T of the soft gluon (LL)
- $\mathcal{O}(\alpha_S^2)$ correction included proportional to the coefficient K

Soft-gluon effective coupling to NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_F$$

Besides application in analytic resummation

- Use in Monte Carlo parton shower (partial inclusion of NLL terms)
- Dispersive approach to power corrections

Dokshitzer, Marchesini, Webber (1996)
Dokshitzer, Lucenti, Marchesini, Salam (1998)

The cusp anomalous dimension

Since the soft coupling controls soft-collinear emissions

➡ We may compare it with soft limit of the DGLAP kernel

$$P_{ii}(\alpha_S; z) = \frac{1}{1-z} A_i(\alpha_S) + \ldots \qquad A_i(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_i^{(n)}$$

Korchemsky (1989)

Controlled by the (light-like) cusp anomalous dimension $A_i(\alpha_S)$

In the $\overline{\mathrm{MS}}$ scheme:

$$A_i^{(1)} = C_i \ ,$$

$$A_i^{(2)} = \frac{1}{2} K C_i \ ,$$

$$A_i^{(3)} = C_i \left[\left(\frac{245}{96} - \frac{67}{216}\pi^2 + \frac{11}{720}\pi^4 + \frac{11}{24}\zeta_3 \right) C_A^2 + \left(-\frac{209}{432} + \frac{5}{108}\pi^2 - \frac{7}{12}\zeta_3 \right) C_A n_F + \left(-\frac{55}{96} + \frac{1}{2}\zeta_3 \right) C_F n_F - \frac{1}{108} n_F^2 \right] ,$$

Moch, Vermaseren, Vogt (2004)

➡ **Up to second order $A_i(\alpha_S)$ coincides with the soft coupling $\mathcal{A}_i^{CMW}$**

The cusp anomalous dimension

One may be tempted to conclude that this relation holds at higher order but…

This cannot hold in general !

Indeed A_i depends on the factorisation scheme while a physical coupling should not

Transverse-momentum integration in the Sudakov kernel does not lead to collinear singularities because regularised by the observable one is interested in

Example: transverse-momentum resummation

$$S_i(b) \sim \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq_t^2}{q_t^2} A_i(\alpha_S(q_t^2)) \ln Q^2/q_t^2 \right\}$$

Lower bound on q_t is of order 1/b

But equality up to $\mathcal{O}(\alpha_S^2)$ not accidental: up to this order cut-off on q_T^2 essentially equivalent to $\overline{\text{MS}}$ factorisation

The soft-coupling to all orders

Consider generic hard process with two-hard partons ($q\bar{q}$ pair or gg)

An all-order definition of the soft coupling can be given starting from the web: $w_i(k,\epsilon)$

Banfi, El-Menoufi, Monni (2018)

Probability of correlated emission of an arbitrary number of soft-collinear partons with total momentum k in $d = 4 - 2\epsilon$ dimensions

$$w_i\left(k;\epsilon\right) \sim \sum_{n=1}^{\infty} \int \left(\prod_{i=1}^{n} \left[dk_i\right] \right) \tilde{M}_s^2\left(k_1, \ldots, k_n\right) (2\pi)^d \delta^{(d)} \left(k - \sum_i k_i \right)$$

Correlated contribution in the soft limit

$$M_s^2(k_1) = \tilde{M}_s^2(k_1)$$

$$M_s^2(k_1,k_2) = \left[\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2)\right]_{\rm sym} + \tilde{M}_s^2(k_1,k_2)$$

$$M_s^2(k_1,k_2,k_3) = \left[\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2)\tilde{M}_s^2(k_3)\right]_{\rm sym} + \left[\left(\tilde{M}_s^2(k_1)\tilde{M}_s^2(k_2,k_3)\right)_{\rm sym} + (1\leftrightarrow 2) + (1\leftrightarrow 3)\right] + \tilde{M}_s^2(k_1,k_2,k_3)$$

The soft-coupling to all orders

The web is boost invariant: can depend only on k_T^2 and $m_T^2 = k^2 + k_T^2$

Two natural definitions of soft-coupling can be provided

- $$\widetilde{\mathcal{A}}_{T,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - k_T^2\right) w_i(k;\epsilon)$$

See also Banfi, El-Menoufi, Monni (2018)

Fix k_T^2 and integrate over m_T^2

↘ suitable for q_T-related observables

- $$\widetilde{\mathcal{A}}_{0,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - m_T^2\right) w_i(k;\epsilon)$$

Fix m_T^2 and integrate over k_T^2

↘ suitable for threshold-related observables

The soft-coupling to all orders

Both couplings can be evaluated at $\epsilon = 0$ (IR singularities cancel in the web) but useful to keep ϵ-dependence

$$\mathscr{A}_i(\alpha_S) = \widetilde{\mathscr{A}}_i(\alpha_S; \epsilon = 0)$$

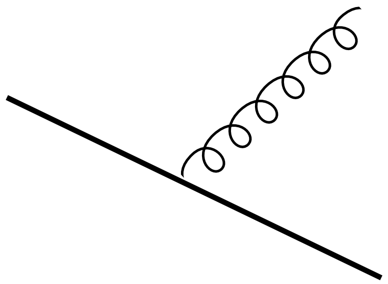
$$\widetilde{\mathscr{A}}_i^{(n)}(\epsilon) = \mathscr{A}_i^{(n)} + \sum_{k=1}^{\infty} \epsilon^k \widetilde{\mathscr{A}}_i^{(n;k)}$$

At the lowest order we simply have $w_i(k, \epsilon) \sim \delta(k^2) = \delta(m_T^2 - k_T^2)$

$$\widetilde{\mathscr{A}}_{T,i}^{(1)}(\epsilon) = \widetilde{\mathscr{A}}_{0,i}^{(1)}(\epsilon) = C_i\, c(\epsilon)$$

where

$$c(\epsilon) \equiv \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} = 1 - \frac{\pi^2}{12}\epsilon^2 - \frac{1}{3}\zeta_3 \epsilon^3 + \mathcal{O}\left(\epsilon^4\right)$$



ϵ-dependence starts at $\mathcal{O}(\epsilon^2)$

We have computed both couplings in $d = 4 - 2\epsilon$ dimensions up to $\mathcal{O}(\alpha_S^2)$

$\mathcal{O}(\alpha_S^2)$ results

Catani, de Florian, MG (2019)
Catani, de Florian, Devoto, Mazzitelli, MG, to appear

$$\widetilde{\mathcal{A}}^{(2)}_{T,i}(\epsilon) = C_i \left\{ -\frac{c(\epsilon)\,(11C_A - 2n_F)}{12\,\epsilon} + \frac{c(2\epsilon)\,\pi}{\sin(\pi\epsilon)} \frac{[C_A(11-7\epsilon) - 2\,n_F(1-\epsilon)]}{4(3-2\epsilon)(1-2\epsilon)} + \frac{C_A\,c(2\epsilon)\,h(\epsilon)\,\pi}{2\sin(\pi\epsilon)} - \frac{C_A\,c(2\epsilon)\,\pi^2}{2\sin^2(\pi\epsilon)} \left(\frac{2-\sin^2(\pi\epsilon)}{\cos(\pi\epsilon)} - \frac{2\sin(\pi\epsilon)}{\pi\epsilon} \right) \right\}$$

where $h(\epsilon) = \gamma_E + \psi(1-\epsilon) + 2\,\psi(1+2\epsilon) - 2\,\psi(1+\epsilon)$

$$\widetilde{\mathcal{A}}^{(2)}_{0,i}(\epsilon) = C_i \left\{ -\frac{c(\epsilon)\,(11C_A - 2n_F)}{12\,\epsilon} + \frac{c^2(2\epsilon)}{\epsilon\,c^2(\epsilon)} \frac{[C_A(11-7\epsilon) - 2\,n_F(1-\epsilon)]}{4(3-2\epsilon)(1-2\epsilon)} + \frac{C_A\,c^2(2\epsilon)\,r(\epsilon)}{2(1-2\epsilon)\,c^2(\epsilon)} - \frac{C_A\,c(2\epsilon)}{2\,\epsilon^2} \left(\frac{(\pi\epsilon)^2\cos(\pi\epsilon)}{\sin^2(\pi\epsilon)} + \frac{\pi\,\epsilon}{\sin(\pi\epsilon)} - \frac{2\,c(2\epsilon)}{c^2(\epsilon)} \right) \right\}$$

where $r(\epsilon) = \dfrac{2}{1+\epsilon}\ {}_3F_2(1,1,1-\epsilon;2-2\epsilon,2+\epsilon;1) - \dfrac{1}{1-\epsilon}\ {}_3F_2(1,1,1-\epsilon;2-2\epsilon,2-\epsilon;1)$

$\mathcal{O}(\alpha_S^2)$ results

Expanding up to $\mathcal{O}(\epsilon^2)$:

$$\widetilde{\mathscr{A}}^{(2)}_{T,i}(\epsilon) = A^{(2)}_i + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{\pi^2}{72} - \frac{14}{27} \right) \right]$$
$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{216} - \frac{77\zeta_3}{36} - \frac{7\pi^4}{120} \right) + n_F \left(\frac{5\pi^2}{108} - \frac{82}{81} + \frac{7\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

$$\widetilde{\mathscr{A}}^{(2)}_{0,i}(\epsilon) = A^{(2)}_i + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{5\pi^2}{72} - \frac{14}{27} \right) \right]$$
$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{72} - \frac{143\zeta_3}{36} - \frac{\pi^4}{36} \right) + n_F \left(\frac{5\pi^2}{36} - \frac{82}{81} + \frac{13\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

The two couplings coincide for $\epsilon = 0$ and agree with the cusp anomalous dimension but they differ starting from $\mathcal{O}(\epsilon)$ (different phase space integration)

Conformal relation

d-dimensional formulation important in view of applications to hadron collisions

➡ Consider the running of the QCD coupling in d-dimensions

$$\frac{d \ln \alpha_s(\mu^2)}{d \ln \mu^2} = -\epsilon + \beta(\alpha_s(\mu^2)) \qquad \beta = -(\beta_0 \alpha_s + \beta_1 \alpha_s^2 + \ldots)$$

The conformal point is defined as the point in which $\epsilon = \beta(\alpha_S)$

We find that the two-soft couplings coincide at the conformal point

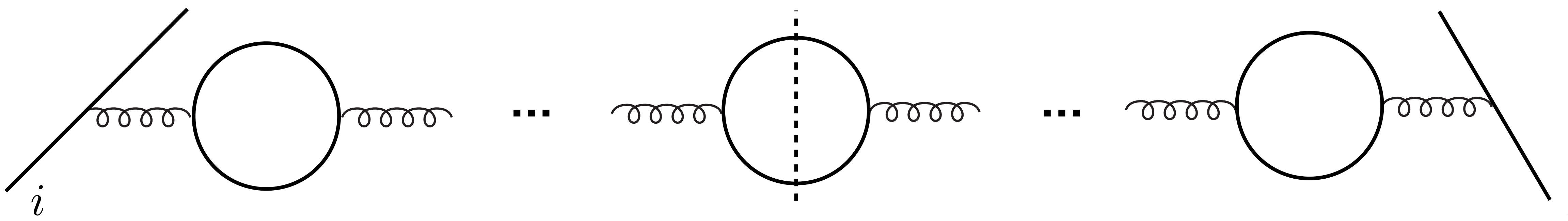
$$\widetilde{\mathscr{A}}_{T,i}\Big(\alpha_S; \epsilon = \beta\left(\alpha_S\right)\Big) = \widetilde{\mathscr{A}}_{0,i}\Big(\alpha_S; \epsilon = \beta\left(\alpha_S\right)\Big) = A_i\left(\alpha_S\right)$$

In short: formulate threshold resummation in d-dimensions, carry out $\overline{\text{MS}}$ factorisation and identify the cusp anomalous dimension

Conformal relation

The conformal relation is consistent with the explicit computation of the leading n_F term at higher orders

Catani, de Florian, Devoto, Mazzitelli, MG, to appear



In this limit we can also sum the series and compare with all-order result from Beneke-Braun

$$A_i = C_i \frac{\alpha_S}{\pi} \frac{\Gamma(4 + 2\beta_0\alpha_S)}{6\Gamma(1-\beta_0\alpha_S)\,\Gamma^2(2+\beta_0\alpha_S)\,\Gamma(1+\beta_0\alpha_S)}$$

✓

Conformal relation at work

The conformal relation can be exploited to obtain further results

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=A_i\left(\alpha_S\right)$$

Expanding to third order we can express the soft coupling in terms of the cusp anomalous dimension and of the ϵ-dependence at lowest order

$$\mathscr{A}_i^{(3)}=A_i^{(3)}-\left(\beta_0\pi\right)^2\widetilde{\mathscr{A}}_i^{(1;2)}+\left(\beta_0\pi\right)\widetilde{\mathscr{A}}_i^{(2;1)}$$

For $\mathscr{A}_{T,i}^{(3)}$ we get

$$\mathscr{A}_{T,i}^{(3)}=A_i^{(3)}+C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12}+C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27}-\frac{11\pi^2}{144}-\frac{7\zeta_3}{2}\right)+n_F\left(\frac{\pi^2}{72}-\frac{14}{27}\right)\right]$$

Agrees with result of Banfi et al and with $A_i^{(3)}$ coefficient for q_T-resummation

Becher, Neuber (2011)

Conformal relation at work

For $\mathscr{A}_{0,i}^{(3)}$ we get

$$\mathscr{A}_{0,i}^{(3)} = A_i^{(3)} + C_i\,(\beta_0\pi)^2\,\frac{\pi^2}{12} + C_i\,(\beta_0\pi)\left[C_A\left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2}\right) + n_F\left(\frac{5\pi^2}{72} - \frac{14}{27}\right)\right]$$

Controls threshold resummation at NNLL accuracy

In the case of threshold resummation we can exploit the available results up to N³LL to obtain

$$\begin{aligned}\mathscr{A}_{0,i}^{(4)} = A_i^{(4)} + C_i \Bigg\{ & C_A^3\left(\frac{121\pi^2\zeta_3}{288} - \frac{21755\zeta_3}{864} + \frac{33\zeta_5}{4} + \frac{33\zeta_5}{17280} - \frac{41525\pi^2}{15552} + \frac{3761815}{186624}\right) \\ & + C_A^2 n_F\left(-\frac{11\pi^2\zeta_3}{144} + \frac{6407\zeta_3}{864} - \frac{3\zeta_5}{2} - \frac{11\pi^4}{432} + \frac{9605\pi^2}{7776} - \frac{15593}{1944}\right) \\ & + C_A C_F n_F\left(\frac{17\zeta_3}{9} + \frac{11\pi^4}{1440} + \frac{55\pi^2}{576} - \frac{7351}{2304}\right) + C_A n_F^2\left(-\frac{179\zeta_3}{432} + \frac{13\pi^4}{4320} - \frac{695\pi^2}{3888}\right) \\ & + C_F n_F^2\left(-\frac{19\zeta_3}{72} - \frac{\pi^4}{720} - \frac{5\pi^2}{288} + \frac{215}{384}\right) + n_F^3\left(-\frac{\zeta_3}{108} + \frac{5\pi^2}{648} - \frac{29}{1458}\right)\Bigg\}\end{aligned}$$

$A_i^{(4)}$ known numerically

Moch, Ruijl, Ueda, Vermaseren, Vogt (2018)

Casimir scaling violation

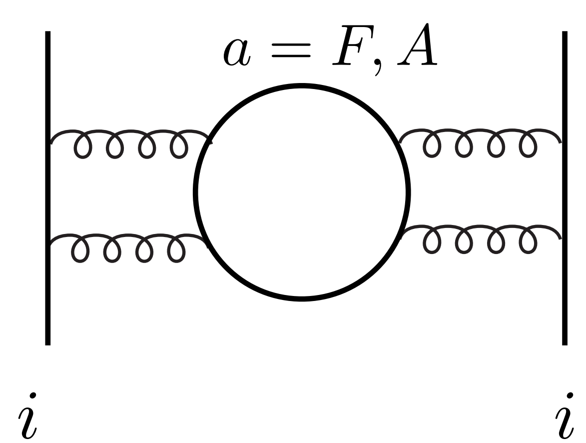
The structure of the fourth-order contributions to the web $w_i(k;\epsilon)$ and to the soft coupling leads to the following colour decomposition

$$\widetilde{\mathscr{A}}_i^{(4)}(\epsilon) = C_i \widetilde{\mathscr{A}}_{[2]}^{(4)}(\epsilon) + \frac{d_{Ai}^{(4)}}{N_i} \widetilde{\mathscr{A}}_{[4A]}^{(4)}(\epsilon) + n_F \frac{d_{Fi}^{(4)}}{N_i} \widetilde{\mathscr{A}}_{[4F]}^{(4)}(\epsilon)$$

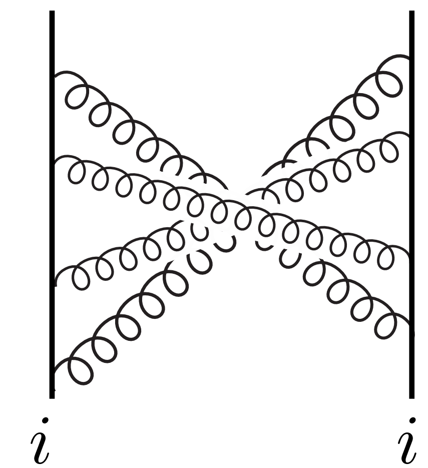
Naive Casimir scaling is violated and quartic Casimir invariants appear

$$d_{xy}^{(4)} = d_x^{abcd} d_y^{abcd} \qquad d_r^{abcd} = \frac{1}{6} \mathrm{Tr}\left(T_r^a T_r^b T_r^c T_r^d + 5 \text{ perm}.\right)$$

Here $N_i = N_c^2 - 1$ if $i = g$ and $N_i = N_c$ if $i = q, \bar{q}$



+ perm.



Related to colour "monster": irreducible $1/N_c^2$ suppressed correlation appearing in the emission of 4 soft gluons

Dokshitzer et al. "Basics of Perturbative QCD"
Catani, Colferai, Torrini (2019)

Casimir scaling violation

The structure of the fourth-order contributions to the web $w_i(k;\epsilon)$ and to the soft coupling leads to the following colour decomposition

$$\widetilde{\mathscr{A}}_i^{(4)}(\epsilon) = C_i \widetilde{\mathscr{A}}_{[2]}^{(4)}(\epsilon) + \frac{d_{Ai}^{(4)}}{N_i} \widetilde{\mathscr{A}}_{[4A]}^{(4)}(\epsilon) + n_F \frac{d_{Fi}^{(4)}}{N_i} \widetilde{\mathscr{A}}_{[4F]}^{(4)}(\epsilon)$$

Naive Casimir scaling is violated and quartic Casimir invariants appear

Entire dependence on i embodied in the colour coefficients

$\widetilde{\mathscr{A}}_{[2]}^{(4)}(\epsilon)$ depends on N_c and n_F while $\widetilde{\mathscr{A}}_{[4A]}^{(4)}(\epsilon)$ and $\widetilde{\mathscr{A}}_{[4F]}^{(4)}(\epsilon)$ do not

➡ **Generalised Casimir scaling**

Casimir scaling violation

Since the difference $\mathscr{A}_{0,i}^{(4)} - A_i^{(4)}$ fulfils ordinary Casimir scaling we conclude that also $A_i^{(4)}$ fulfils generalised scaling

$$A_i^{(4)} = C_i\, A_{[2]}^{(4)} + \frac{d_{Ai}^{(4)}}{N_i} A_{[4A]}^{(4)} + n_F \frac{d_{Fi}^{(4)}}{N_i} A_{[4F]}^{(4)}$$

Conjectured and verified numerically by Moch et al. in arXiv 1805.09638

The coefficient $A_{[4F]}^{(4)}$ has been recently computed analytically

Lee, Smirnov & Smirnov, Steinhauser (2019)
Henn, Peraro, Stahlhofen, Wasser (2019)

Full n_F dependence of $A_i^{(4)}$ now known analytically

Beneke, Braun (1995)
Grozin, Henn, Korchemsky, Marquard (2015)
Davies, Vogt, Ruijl, Ueda, Vermaseren (2016)

Further comments

Due to the numerical dominance of the extra term the soft coupling $\mathcal{A}_{0,i}$ is known almost exactly up to the fourth order $(N_c = 3,\, n_F = 5)$

$$\mathcal{A}_{0,i}\left(\alpha_S\right) = C_i \frac{\alpha_S}{\pi}\left[1 + 0.54973\alpha_S - 1.7157\alpha_S^2 - \left(5.9803(3)\delta_{iq} + 6.236(2)\delta_{ig}\right)\alpha_S^3 + \mathcal{O}\left(\alpha_S^4\right)\right]$$

In general the Sudakov kernel has other components that need to be taken into account:

- Soft emissions at large angles
- Collinear (non soft) emissions

But the soft-collinear component is enhanced

→ at N^{k}LL we need the Sudakov kernel at $\mathcal{O}(\alpha_S^{k+1})$ and the other components at $\mathcal{O}(\alpha_S^k)$

Summary

- The soft-gluon effective coupling controls the intensity of soft and collinear radiation in the Sudakov kernel

- We provided two possible definitions of the soft-gluon effective coupling $\widetilde{\mathscr{A}}_{T,i}$ and $\widetilde{\mathscr{A}}_{0,i}$ to all perturbative orders

- At the conformal point $\epsilon = \beta(\alpha_S)$ the two coupling coincide and are equal to the cusp anomalous dimension

$$\widetilde{\mathscr{A}}_{T,i}\Big(\alpha_S; \epsilon = \beta\left(\alpha_S\right)\Big) = \widetilde{\mathscr{A}}_{0,i}\Big(\alpha_S; \epsilon = \beta\left(\alpha_S\right)\Big) = A_i\left(\alpha_S\right)$$

- Consistent with all-order result for the cusp anomalous dimension in the large-n_F limit

Summary

- We have explicitly computed the two couplings at $\mathcal{O}(\alpha_S^2)$ in d dimensions
- This result allows us to extract the four dimensional expressions of the two soft couplings at third order
- In the case of $\mathscr{A}_{T,i}$ the result coincides with the corresponding coefficient for q_T resummation
- In the case of $\mathscr{A}_{0,i}$ we can obtain the relation with the cusp anomalous dimension to the fourth order (relevant for N^3LL resummation)

Thank you for your attention !